Kinetic friction due to Ohm's law heating

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Abstract
Using both a recent calculation by Bruch of the damping of the motion of a
monolayer nitrogen film oscillating harmonically on a metallic surface due to
Ohm's law heating and a Thomas–Fermi approximation treatment of the Ohm's
law heating mechanism, which accounts for the nonzero thickness of the surface
region of a metal, it is argued that this mechanism for friction is able to account
for recent measurements of the drop in the friction for a nitrogen film sliding
over a lead substrate as it goes below its superconducting transition temperature.
Bruch's calculation is also made more transparent by redoing the calculation
for a film sliding at constant speed, instead of oscillating. Using this treatment,
it is easily shown that Bruch's calculation is equivalent to integrating Boyer's
solution of the problem of a charge sliding over a metallic surface over the
charge density of the monolayer nitrogen film.

1. Introduction

Motivated by attempts [1–3] to explain a recent quartz crystal microbalance experiment [4]
which shows a rapid drop in the friction of a film of nitrogen molecules sliding on a lead
substrate, on dropping below the superconducting transition temperature T_c of the substrate,
Bruch has recently done a calculation of the electronic contribution to the friction for a
monolayer nitrogen film executing simple harmonic motion on a metallic substrate [5]. In
contrast to calculations of a single molecule moving on the substrate [6, 8], which require that
the molecule possess a larger dipole moment or charge than is generally accepted for adsorbed
molecules, in order to explain the experimental results [4], Bruch's results suggest that the
field due to the quadrupole moment of the nitrogen molecule can explain the microbalance
experiments [5] if the molecules form a monolayer film. The reason for this is that for
a monolayer film the field inside the metallic substrate on which the film is moving falls
exponentially to zero over a distance of the order of a lattice constant of the film below the
surface of the metal. Since the distance over which the field is nonzero is much smaller than a
mean free path, Bruch pointed out that the problem must be treated in the anomalous skin effect
regime [7], in which only those electrons with velocities nearly parallel to the film remain in
this region for a sufficient length of time to be significantly accelerated by the electric field.

Consequently, only a fraction $(G\ell)^{-1}$ of the electrons (where G is the magnitude of a typical reciprocal-lattice vector and ℓ is the mean free path) can be accelerated by the field. This results in an enhancement of the effective resistivity of the skin depth region, which leads to an enhancement of the rate of dissipation and the contribution to the kinetic friction due to Ohm's law heating.

In addition to the Ohm's law heating mechanism for electronic friction put forward in [1,2] as a possible way of explaining the experimental results reported in [4], there is another mechanism for electronic friction (actually, the most commonly suggested mechanism for this phenomenon) which is due to the creation of electron–hole pairs of nonzero energy [9–12]. The physical difference between these two mechanisms can be understood as follows: in the electron–hole pair mechanism for electronic friction, the energy loss due to friction is ascribed to the energy needed to create the electron–hole pairs of nonzero energy. In contrast, in the Ohm's law heating mechanism considered in [1, 2, 5] and in this article, the energy loss need not be due to the energy cost necessary to create electron–hole pairs of nonzero energy. In this mechanism, if we consider the case in which the film is slid along at constant speed (e.g., by an applied force), the electric field resulting from the sliding film results in a screening charge near the surface of the metal, which is dragged along with the film. This results in an electric current. Let us first consider only elastic scattering of the electrons (the dominant contribution to the resistivity well below the Debye temperature). During the sliding, electrons get scattered elastically by impurities and other defects. This would result in a reduction in the drift velocity, and hence the electric current, except that we force the current to remain constant by forcing the film to move at constant speed. In order to maintain the current, the electric field acting on the conduction electrons due to the film must accelerate them in order to maintain the drift velocity. The work done by this field is identified with the contribution to the dissipation produced by Ohm's law heating. In the Ohm's law heating mechanism considering only elastic scattering of the electrons, the electron–hole pairs resulting from the scattering of the electrons by defects in the substrate have zero energy because the scattering is elastic. In addition to the contribution due to elastic scattering, there is also inelastic scattering of the electrons by phonons, which results in an additional energy loss from energy transferred from the electrons to the phonons. Below T_c, the above scattering mechanisms do not occur for the superconducting electrons because of the gap in their excitation spectrum. The screening charge will be transported entirely by the superconducting electrons because they can flow without electrical resistance, and hence are able to short circuit the current due to the normal electrons. Since the superconducting electrons are not scattered, the Ohm's law heating mechanism for dissipation (and hence kinetic friction) described above does not operate.

Persson [13] has argued that the electron–hole pair mechanism should dominate over the Ohm's law heating mechanism by three orders of magnitude for a charged ion moving above the surface of a metallic substrate. Since this mechanism depends on the density of normal electrons, which does not drop rapidly on falling below the superconducting transition temperature, it cannot account for the experimental result reported in [4]. The calculation of the Ohm's law heating contribution of the friction, including the anomalous skin effect conductivity, as suggested by Bruch [5], however, can be of the same order of magnitude as the electron–hole contribution to the friction or greater. Since the resistivity drops to zero over a relatively small temperature range on dropping below T_c, the latter mechanism for electronic friction drops rapidly in much the same way as in [4], and, as we shall see, it is large enough to explain the results reported in [4]. As pointed out at the end of section 4 in this paper, however, resolution of the question of how to understand the experiment reported in [4] will only be complete when the surfaces used in the experiment have been better characterized.

In Bruch's treatment, the simple harmonic motion of the film results in an electric field with many harmonics of the frequency of the film's oscillations. In contrast, if one considers the film to be sliding over the substrate at constant speed, the field will possess only a single harmonic of the 'washboard frequency', which is the characteristic frequency in this formulation of the problem. In section 2, Bruch's calculation is reformulated by assuming that the film slides at constant velocity, rather than executing simple harmonic motion. It is expected that this will give the same value for the friction, and it simplifies the calculation. It is argued in section 3 that this version of Bruch's treatment is identical to Boyer's treatment [6], which is identical to the treatment of the problem due to Tomassone and Widom [8], used in [1,2]. Since [13] stresses the necessity of taking into account the nonzero thickness of the surface region, and since Bruch's work is a classical treatment of the surface of a metal (i.e., one which assumes that the surface region has zero thickness), in section 4 a calculation is presented which takes into account the fact that the surface region, when treated quantum mechanically, has a nonzero thickness, in contrast to the zero thickness that it has in treatments of this problem using classical electrodynamics [1–3, 5–8].

2. Bruch's idea applied to a uniformly sliding film

Let us take the region with $z < 0$ to be occupied by the metal, with the region with $z > 0$ occupied by free space. Bruch [5] writes the z-component of the electric field in terms of its Fourier transform in the x- and y-coordinates, $E(G, z)$:

$$E_z(r) = \sum_G E(G, z) e^{iG \cdot r}, \tag{1}$$

where G denotes a reciprocal-lattice vector of the nitrogen film. Using the requirement that the field have zero divergence (in regions with zero charge density), the field components parallel to the surface are given by

$$E_\parallel(r) = \sum_G (iG/G^2)(\partial E(G, z)/\partial z) e^{iG \cdot r}. \tag{2}$$

In regions in which there is no net charge density the current density $J(r)$ can similarly be expressed in terms of the Fourier transform of its z-component. In Bruch's work, the film is assumed to execute simple harmonic motion of frequency Ω as a rigid unit, which means that the time-dependent fields are obtained by replacing r by $r - A \cos(\Omega t)$, where the vector A has magnitude equal to the amplitude and direction in the direction of motion of the film. When this substitution is made in equations (1) and (2), the time dependence of the fields is a sum of harmonics $K\Omega$, where K is an integer, with Bessel function coefficients $J_K(G \cdot A)$. In Bruch's solution it is necessary to deal with all of the time Fourier components. In this section, I propose that the inverse slip-time, which Bruch obtains, can be obtained much more easily by considering the damping of a film moving at constant velocity v instead. For the present treatment, in which the film slides at constant velocity v, we obtain the time dependence simply by replacing r by $r - vt$, which results in a field which contains only a single time Fourier component for each reciprocal-lattice vector, with frequency equal to the 'washboard frequency', $G \cdot v$, and there is no sum over harmonics with Bessel function coefficients. Because the inverse slip-time only depends on Ohm's law heating due to the flow of the screening charge in the metal, this method is expected to give the same value for the slip-time as Bruch obtains. Justification for this is given at the end of section 3. Bruch obtains a relationship between the time and space Fourier transforms of the z-component of the current density inside the film, and the z-component of the time and space Fourier component of the field on the surface of the metal, just inside the metal (equation (2.10) in Bruch's paper)

by solving the linearized Boltzmann equation simultaneously with Faraday's and Ampere's laws. Because the frequencies involved in this problem are quite small, Bruch solves these equations in the zero-frequency limit. As a consequence, each time Fourier component of the field and the current density satisfies the same equations (since in the zero-frequency limit, the coefficients multiplying the fields in the equations are obviously independent of frequency). Thus, Bruch's solution of Boltzmann's equation with Faraday's and Ampere's law can be equally well applied to the present case of a film sliding at constant velocity, for which there is only one time Fourier component for each reciprocal-lattice vector G, e.g., $E(G, z)$ for the z-component of the electric field. One obtains for the relationship between the Fourier transforms of the z-component of the current density and field just below the surface of the metal

$$J(G, z = 0^-) = \sigma_G E(G, z = 0^-), \tag{3}$$

where $\sigma_G = 3\sigma(1-p)/(4G\ell)$, where ℓ is the mean free path, p is the fraction of the conduction electrons which are specularly reflected at the surface of the metal at $z = 0$, and σ is the Ohm's law electrical conductivity. This is Bruch's equation (2.10). Substituting equation (3) in the standard boundary condition [14]

$$J_z(r, z = 0^-) = (4\pi)^{-1} \frac{\partial}{\partial t}[E_z(r, z = 0^-) - E_z(r, z = 0^+)], \tag{4}$$

we obtain

$$\sum_G \sigma_G E(G, z = 0^-)e^{iG \cdot r} = (4\pi)^{-1} \partial[E_z(z = 0^-) - E_z(z = 0^+)]/\partial t$$

$$= i(4\pi)^{-1} \sum_G (G \cdot v)[E(G, z = 0^-) - E(G, z = 0^+)]e^{iG \cdot r}, \tag{5}$$

where we have used the fact that E and J have the time dependence $e^{-iG \cdot vt}$ for a film sliding at constant speed, for the reasons given above. Equation (5) can be written as

$$(1 + i\lambda_G)E(G, 0^-) = E(G, 0^+) = B_G e^{Gz}|_{z=0+} + E_i(G, 0^+), \tag{6}$$

where $E_i(G, z)$ and $E(G, z)$ are the Fourier transforms of the z-component of the contribution to the field from the film in the absence of the substrate and that inside the metal, respectively, and $B_G e^{Gz}$ is the contribution of the Fourier transform of the z-component of the field outside of the substrate due to the charge density induced by the film. B_G is a constant to be determined by the boundary conditions. The parameter $\lambda_G = 4\pi\sigma_G/(G \cdot v)$, where $(G \cdot v)$ is the 'washboard' frequency of the film. In addition to equation (6), we must require continuity of the component of the field parallel to the surface, which is given by

$$dE(G, z)/dz|_{z=0^-} = [d(B_G e^{Gz})/dz + dE_i(G, z)/dz]|_{z=0^+}, \tag{7}$$

which, using the result

$$E(G, z) = E(G, z = 0^-)e^{Gz}$$

given in equation (A.13) of the appendix of [5] (it follows from the solution of Ampere's and Faraday's law in the zero-frequency limit), gives

$$-E(G, 0^+) = B_G - E_i(G, 0^-). \tag{8}$$

The solution to equations (6) and (8) is

$$E(G, 0^+) = [2/(2 + i\lambda_G)]E_i(G, 0^-), \tag{9a}$$

and

$$B_G = [i\lambda_G/(2 + i\lambda_G)]E_i(G, 0^-) \approx (1 + 2i/\lambda_G + \cdots)E_i(G, 0^-), \tag{9b}$$

where $E_i(G, Z)$ can be found using

$$E_i(r) = \int d^3r' \, \rho(r') \frac{r - r'}{|r - r'|^3}, \tag{10}$$

where $\rho(r)$ is the film's charge density. It can be crudely modelled by three charges along the axis of a molecule ℓ_β, in the small $-\ell_\beta$ limit: two charges of charge $+q$ at the outer edges of the molecule and a charge $-2q$ at its centre with q chosen so as to give the experimental value of the quadrupole moment θ. Then,

$$\rho(r) = q\delta(z + h) \sum_{R_j, \beta = 1,2} [\delta^{(2)}(r - R_j - \rho_\beta - \ell_\beta) + \delta^{(2)}(r - R_j - \rho_\beta + \ell_\beta)$$
$$- 2\delta^{(2)}(r - R_j - \rho_\beta)], \tag{11}$$

where $\delta^{(2)}$ denotes a two-dimensional delta function, ρ_β denotes the position of a molecule in the unit cell, and R_j is the location of the origin of the jth unit cell. The choice of having the three charges lie in a plane parallel to the surface simulates the low-temperature arrangement of the nitrogen quadrupole in monolayer films at the low temperatures of the experiment. Writing

$$\frac{r - r'}{|r - r'|^3} = 4\pi(2\pi)^{-3} \int d^3k \, (ik/k^2) e^{ik \cdot (r - r')}$$

and substituting in equation (10), we obtain

$$E_i(G, 0^-) = -(\pi\theta/A_c) e^{-Gh} \sum_\beta e^{-iG \cdot \rho_\beta} (G \cdot \hat{\ell}_\beta)^2, \tag{12}$$

where ρ_β is the location of the βth molecule in the nitrogen film unit cell, $\hat{\ell}_\beta$ is the symmetry axis of βth molecule, and θ is the quadrupole moment of a single molecule. The inverse slip-time found by calculating the force exerted on the film by the image field outside the metal,

$$\sum_G (iG/G^2) \, \partial[B_G e^{Gz} e^{iG \cdot r}]/\partial z,$$

is given by $(NMv)^{-1}$ times the force or

$$(NMv)^{-1} \int d^3r \, \rho(r) \sum_G (iG/G^2) \, \partial[B e^{Gz} e^{iG \cdot r}]/\partial z. \tag{13}$$

From equations (9b) and (13), we obtain for the inverse slip-time

$$\tau^{-1} = \left(\frac{\theta^2}{3(1-p)\sigma A_c M v}\right) \left| \sum_G e^{-2Gh}(4G\ell)(G \cdot v)(G/G) \left| \sum_\beta (G \cdot \hat{\ell}_\beta)^2 e^{iG \cdot \rho_\beta} \right|^2 \right|, \tag{14}$$

which gives

$$\tau^{-1} = \left(\frac{\theta^2}{3(1-p)\sigma A_c M}\right) \sum_G (4G\ell)(G_x^2/G) e^{-2Gh} \left| \sum_\beta (G \cdot \hat{\ell}_\beta)^2 e^{iG \cdot \rho_\beta} \right|^2, \tag{15}$$

where we have taken v to be along the x-axis. By energy conservation, this expression for τ^{-1} must be equal to the value calculated by setting the power loss due to Ohm's law heating equal to NMv^2/τ, the rate at which the viscous electronic contribution to the friction force, NMv/τ, does work on the film. Using the method used in this paper of assuming that the film is sliding at a speed v, rather than oscillating (as assumed in Bruch's paper), the Ohm's law heating power is given by

$$P = 2NA_c \sum_G G^{-1} |E_i(G, 0^-)|^2 (G \cdot v)/\sigma_G. \tag{16}$$

The factor NA_c appears because when P is calculated by integrating $\boldsymbol{J}(\boldsymbol{r}) \cdot \boldsymbol{E}(\boldsymbol{r})$ over the volume of the metal, the volume integral involves evaluating the integral $\int \mathrm{d}^2r\, \mathrm{e}^{\mathrm{i}(G-G')\cdot \boldsymbol{r}} = NA_c\delta_{G,G'}$. We obtain with this procedure[1], substituting E_i from equation (12) in (16), the value for τ^{-1} obtained in equation (15).

3. Treatment of electronic friction using Boyer's solution

Boyer [6] solves the problem of a charge or electric dipole moving above (i.e., outside the metal) and parallel to the surface of the metal. He solves the electrodynamics problem subject to the same boundary conditions as Bruch uses [14], (equation (4) above), which for Boyer's problem is written as

$$J_z = \sigma E = (4\pi)^{-1}\partial E_z/\partial t = (4\pi)^{-1}\boldsymbol{v} \cdot \boldsymbol{\nabla} E_z. \tag{17}$$

When terms of up to first order in v are kept, we obtain Boyer's result for the electric field. The force exerted by this field on the moving charge gives the force of friction due to Ohm's law heating in the metal. The force of friction found by the formalism due to Tomassone and Widom [8] gives the same friction and hence is believed to be equivalent to Boyer's calculation.

Boyer [6] finds, in addition to the electrostatic field, a contribution to the electric field above the substrate linear in the velocity v of a point charge q sliding above the substrate, given by

$$\boldsymbol{E}(\boldsymbol{r}) = -(qv/2\pi\sigma)\frac{\partial}{\partial x}\left[\frac{\boldsymbol{r}-\boldsymbol{r}'}{|\boldsymbol{r}-\boldsymbol{r}'|^3}\right], \tag{18}$$

assuming that the sliding velocity is in the x-direction, where $\boldsymbol{r}' = vt\hat{x}+h\hat{z}$ is the location of the moving charge (where $\hat{x}$ and $\hat{z}$ are unit vectors in the x- and z-directions, respectively). In order to apply this result to a monolayer film of charge density $\rho(\boldsymbol{r})$, let us multiply equation (18) by $\rho(\boldsymbol{r})$ and integrate over volume to obtain $\boldsymbol{E}(\boldsymbol{r})$:

$$\boldsymbol{E}(\boldsymbol{r}) = (v/2\pi\sigma)\int \mathrm{d}^3r'\, \rho(\boldsymbol{r}')(4\pi)(2\pi)^{-3}\int \mathrm{d}^3k\, (\boldsymbol{k}k_x/k^2)\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')} \tag{19}$$

where we have written the field in equation (19) in terms of its Fourier transform. For a periodic monolayer film a height h above the surface of the substrate, $\rho(\boldsymbol{r})$ has the form

$$\rho(\boldsymbol{r}) = \delta(z-h)\sum_G \rho_G \mathrm{e}^{\mathrm{i}G\cdot(\boldsymbol{r}-vt\hat{x})}, \tag{20}$$

where the $\boldsymbol{G}$ are the reciprocal-lattice vectors of the film and $\rho_G = A_c^{-1}\int_u \mathrm{d}^2r\, \rho_2(\boldsymbol{r})\mathrm{e}^{-\mathrm{i}G\cdot \boldsymbol{r}}$, where the u on the integral sign signifies an integral over a unit cell of the film, A_c is the unit-cell area, and $\rho_2(\boldsymbol{r})$ is the charge per unit area of the film. For simplicity, we are modelling the film by a collection of point charges. If the nonzero size of the charges in the film were taken into account, there would be form factors introduced in the summations over $\boldsymbol{G}$, which would fall off rapidly with increasing magnitude of $\boldsymbol{G}$. This can be approximately accounted for by including only the first one or two terms in the sums. Substituting equations (20) in equation (19) gives

$$\boldsymbol{E} = 2v(2\pi\sigma)^{-1}\sum_G \rho_G \int \mathrm{d}k_z \left(\frac{\boldsymbol{k}G_x}{k_z^2+G^2}\right)\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot[\boldsymbol{r}_\parallel-vt\hat{x}-(z+h)\hat{z}]}, \tag{21}$$

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[1] The ångström-scale periodic spatial variation of the field of the sliding film should not make Ohm's law heating inapplicable. We can see this by making a Galilean transformation to a reference frame in which the film is at rest. The Ohm's law heating comes about in this frame from the potential from the impurities in the substrate, which (in this frame) is time dependent since the impurities are moving, and thus can excite conduction electrons near the Fermi level. It is not prevented from doing so by the periodic potential of the film (parallel to the surface) because the periodic field cannot produce gaps which wipe out the entire Fermi surface.

where $r_\parallel$ is the projection of r in the plane of the film, and where the x- and y-components of k are equal to the x- and y-components of the film reciprocal-lattice vectors. For components of E parallel to the substrate, we obtain on performing the integral over k_z

$$E(r) = (v/\sigma) \sum_G \rho_G (G G_x/G) e^{-G(z+h)} e^{i(G \cdot r + G_x vt)}. \tag{22}$$

To find ρ_G, we model the charge density of each molecule by three charges as was done in the last section. Then, substituting $\rho(r)$ from equation (11) in the integral for ρ_G under equation (20), we obtain

$$\rho_G = -4q A_c^{-1} \sum_\beta \sin^2(G \cdot \ell_\beta/2) e^{iG \cdot \rho_\beta}. \tag{23}$$

Therefore,

$$E(r) = -v \, 4q A_c^{-1} \sigma^{-1} \sum_{\beta,G} \sin^2(G \cdot \ell/2)(G G_x/G) e^{-G(z+h)} e^{i[G \cdot (\rho_\beta + r) - G_x vt]}, \tag{24}$$

which reduces to an expression resembling Bruch's for the field in the small-ℓ_β limit with the quadrupole moment of the molecule θ equal to $q\ell_\beta^2$, if we pretend that the system is not in the anomalous skin effect regime, and replace σ_G by σ for the purposes of making a comparison. The force of friction acting on the film is given by

$$F = (1/2) \operatorname{Re} \int d^3r \, \rho^*(r) E(r) = -(N\theta^2 v/\sigma) A_c^{-1} \sum_G \left| \sum_\beta (G \cdot \hat{\ell}_\beta)^2 e^{iG \cdot \rho_\beta} \right|^2 (G G_x/G), \tag{25}$$

where N is the number of molecules in the film, and thus the inverse slip-time τ^{-1} is given by

$$\tau^{-1} = (F/NMv) = A_c^{-1}(\theta^2/M\sigma) \sum_G \left| \sum_\beta (G \cdot \hat{\ell}_\beta)^2 e^{iG \cdot \rho_\beta} \right|^2 (G_x^2/G) e^{-2Gh}, \tag{26}$$

where M is the mass of an adsorbed molecule and $\hat{\ell}_\beta$ is a unit vector along ℓ_β. If as discussed above, we replace σ_G of section 2 by σ, equation (15) becomes identical to equation (26). This demonstrates the equivalence of Bruch's and Boyer's treatments.

Let us now use the methods of this section to present arguments for why one expects the slip-time for an oscillating film (as in Bruch's work) and that of a film sliding at constant speed (as in the present work) to be the same. Note that equation (25) is independent of time. If we treated the problem with an oscillating film, a Bruch did, equation (20) would be replaced by

$$\rho(r) = \delta(z - h) \sum_G \rho_G e^{iG \cdot (r - A \cos(\omega t))}. $$

When this expression for $\rho(r)$ is substituted in equations (19) and (25) and the integrals over r and r' are performed, the $A \cos(\omega t)$ term in the exponent cancels, and hence the only time dependence that survives comes from the fact that v now depends on time. The velocity cancels out, when we divide by v in equation (26) to obtain τ, giving the same results as were obtained for uniform motion.

4. A treatment of the problem which includes the nonzero width of the surface region

Both Bruch's treatment of the problem and Boyer's are based on the classical model for a metallic surface, in which the electronic charge density drops to zero immediately on leaving the metal. In a quantum mechanical treatment, in contrast, the electronic charge density drops to zero over a distance of the order of a couple of ångströms. The film almost certainly resides

in a region just outside the bulk of the metal, in which the electronic charge density is nonzero (although it is decaying exponentially here). Ying *et al* [15], studied electron screening in the surface region using the Thomas–Fermi approximation. They find that a charge placed at the surface will be screened, with a screening length not much longer than that in the bulk metal. The bulk metal Thomas–Fermi expression for the screening charge density of a point charge can be used to model the charge density in the surface region analytically if one fits the screening length to that found in [15]. The bulk Thomas expression for the screening charge density for a point charge is

$$\rho_s(\mathbf{r}) = -(4\pi)^{-1} q k_s^2 \mathrm{e}^{-k_s r}/r = -q k_s^2 (2\pi)^{-3} \int \mathrm{d}^3 k \, \frac{1}{k^2 + k_s^2} \mathrm{e}^{\mathrm{i}k\cdot r} \tag{27}$$

where q is the point charge whose screening is being considered and k_s is the inverse Thomas–Fermi screening length, which we will take here to be a parameter to be fitted to the Ying *et al* calculations [15]. In [15], the quantity

$$\int \mathrm{d}x \, \mathrm{d}y \, \rho(\mathbf{r}) \tag{28}$$

is plotted as a function of z. The model of equation (27) gives for this quantity

$$-(1/2)q k_s \mathrm{e}^{-k_s|z|}, \tag{29}$$

which strongly resembles the quantity plotted in figure 1 of [15]. Then to fit the present approximate model to the results of [15], we can simply choose a value of k_s for which equation (26) reproduces each of the plots in figure 1 of that reference. This method is rigorous when one can use the quasiclassical approximation, which is accurate if typical values of the screening length are much smaller than the thickness of the surface region (see the appendix). When this is not an appropriate limit, for example if the film is further out from the bulk of the metal, there is asymmetry in the screening charge density [16], but it is not such an extreme asymmetry as to invalidate using a spherically symmetric screening charge density, as is done here as a first approximation.

On the basis of this model, we conclude that when a film with charge density $\rho(\mathbf{r})$ moves along the surface of the metal with a velocity $\mathbf{v}$, the screening charge density given by

$$\rho_s(\mathbf{r}) = -(2\pi)^{-3} k_s^2 \int \mathrm{d}^3 r' \int \mathrm{d}^3 k \, \frac{\mathrm{e}^{\mathrm{i}k\cdot(\mathbf{r}-\mathbf{r}')}}{k^2 + k_s^2} \rho(\mathbf{r}') \tag{30}$$

moves with the same velocity, resulting in a current density $\mathbf{J}_s = \mathbf{v}\rho_s(\mathbf{r})$. Substituting for the $\rho(\mathbf{r}')$ in terms of its Fourier transform, we obtain

$$\mathbf{J}_s(\mathbf{r}) = k_s^2 (2\pi)^{-1} \mathbf{v} \sum_G \int \mathrm{d}k_z \, \frac{\mathrm{e}^{\mathrm{i}G\cdot r} \mathrm{e}^{\mathrm{i}k_z(z-h)}}{G^2 + k_s^2 + k_z^2} \rho_G$$

$$= (1/2) k_s^2 \mathbf{v} A_c^{-1} \sum_G \mathrm{e}^{\mathrm{i}G\cdot r} \frac{\exp[-(G^2 + k_s^2)^{1/2}|z - h|]}{(G^2 + k_s^2)^{1/2}} \rho_G. \tag{31}$$

The resulting ohmic heating contribution to the force of friction can be found from

$$F_{fric} v = \sigma^{-1} \int |\mathbf{J}_s|^2 \, \mathrm{d}^3 r. \tag{32}$$

Substituting equation (31) into (32) while replacing σ by an effective conductivity $\sigma_G \approx \sigma/G\ell$ to account for the anomalous skin effect [7] (like the effective conductivity used in section 2) and placing it within the summation over G, we obtain for the force of friction

$$\mathbf{F}_{fric} = A k_s^4 \mathbf{v} \sum_G \sigma_G^{-1} (G^2 + k_s^2)^{-3/2} |\rho_G|^2. \tag{33}$$

Then, we have for the inverse slip-time

$$\tau^{-1} = \frac{F_{fric}}{NMv} = \frac{k_s^4 \theta^2}{2A_c M} \sum_G \sigma_G^{-1} (G^2 + k_s^2)^{-3/2} \left| \sum_\beta (G \cdot \hat{\ell}_\beta)^2 e^{iG \cdot \rho_\beta} \right|^2, \qquad (34)$$

where we have substituted for ρ_G using equation (23), where A is the area of the film, A_c is the area of a unit cell, and N is the number of unit cells in the film. Although the factor of k_s^4 in equation (34) appears to imply that τ^{-1} is a very sensitive function of k_s, it should be noted that k_s^4 is proportional to only the 2/3 power of the electron density. For $k_s \approx 1.95 \times 10^8$ cm^{-1} (the value obtained using parameters appropriate for lead) and $\theta \approx 10^{-26}$ esu, we obtain $\tau^{-1} = 1.05 \times 10^{11}$ s^{-1}, which is at least as large as the electron–hole mechanism [13]. The quantity τ^{-1} found from equation (34) is about three orders of magnitude larger than τ^{-1} found from equation (15) because equation (34) does not contain the factor e^{-2Gh}, which appears in equation (22) (where $G = 2$ Å^{-1} and $h = 2$ Å). In order to obtain the experimental value [4] of τ^{-1}, we would have to use a value for k_s a factor of 5 smaller.

Bruch's use of a classical (i.e., zero-thickness) surface would be valid if the film resided sufficiently far above the surface that the (classical) method of electrical images should be valid [16]. The treatment in this section assumes that the film resides in a part of the surface region in which the electron density is closer to its bulk value. The two treatments bracket the true situation, in which the film lies between these two extremes. Since both treatments give a large enough magnitude for τ^{-1} to account for the experimental results, one can say with confidence that the Ohm's law heating contribution to the friction is of sufficiently large magnitude to account for the experimental results of [4], as put forward in [1,2]. The discussion in this section applies only to a bare metallic substrate surface. There has been some discussion, however, of the possibility that exposure of the lead surface to air in doing the microbalance experiment of [4] could result in the surface having an oxide coating [17]. This might make Bruch's picture of the film residing outside the metal a more accurate description of the situation in the experiment, provided that the oxide layer were not too thick. A complete resolution of the question of how to understand the experiments of [4], however, will have to wait until the surfaces used in the experiment are better characterized.

5. Conclusions

Bruch's calculation of the electronic friction acting on a film of nitrogen molecules harmonically oscillating on a metallic substrate is redone for the simpler case in which the film is sliding at constant speed. This treatment, which should give the same value for the slip-time as Bruch's treatment, clarifies Bruch's treatment and allows one to easily demonstrate that the force of friction found by calculating the force of the image charge acting back on the film and by calculating the Ohm's law heating inside the metallic substrate are equal. It also allows one to demonstrate the equivalence of Bruch's treatment with that due to Boyer of a charge sliding over a metallic surface. Since Boyer's treatment is equivalent to that used in [1,2], one is confident in saying that Bruch's treatment of the problem is an extension of these methods, which allows one to include the anomalous skin effect.

Persson [13] has argued that the contribution to the friction due to the creation of electron–hole pairs should dominate over the Ohm's law heating mechanism by three orders of magnitude for a charged ion moving above the surface of a metallic substrate. Since this mechanism depends on the density of normal electrons, which does not drop rapidly on falling below the superconducting transition temperature, it cannot account for the experimental result reported in [4]. The calculation of the Ohm's law heating contribution of the friction presented in the previous sections, including the anomalous skin effect conductivity, as suggested by Bruch [5],

however, can be of the same order of magnitude as the electron–hole contribution to the friction. Since the resistivity drops to zero over a relatively small temperature range on dropping below T_c, the latter mechanism for electronic friction drops rapidly in much the same way as in [4], and, as we have seen, it is large enough to explain the experimental observations.

Acknowledgments

Appendix. Quasiclassical treatment of the screening of a charge in the surface region

In the surface region, where the electron charge density is decreasing from its bulk value down to zero, the wavefunctions in the jellium model must take the form

$$\psi_k(r) = e^{ik \cdot r} f_k(z), \tag{A.1}$$

where $f_k(z)$ drops from the value that it has for z in the bulk region to zero for z well above the surface. Then the electron charge density is equal to

$$\sum_k n_k |\psi_k(r)|^2 = \sum_k n_k |f_k(z)|^2, \tag{A.2}$$

where n_k is the Fermi function, $(e^{(\epsilon(k)-\mu)/k_B T} + 1)^{-1}$, where $\epsilon(k)$ is the electron energy of wavevector k and μ is the chemical potential. The potential $\phi(r)$ satisfies Poisson's equation:

$$\nabla^2 \phi(r) = -4\pi \sum_k n_k |f_k(z)|^2. \tag{A.3}$$

Let us now consider the screening of a point charge q placed at a point $r = z_0 \hat{z}$ inside the surface region, where $\hat{z}$ is a unit vector in the z-direction. In the linearized Thomas–Fermi treatment of screening, we assume that $\phi(r)$ is changed by a small amount $\delta\phi(r)$ because of the point charge, add this change in the potential to μ, and linearize in $\delta\phi$. Carrying this out we find that $\delta\phi(r)$ satisfies

$$\nabla^2 \delta\phi(r) = -k_s^2(z)\,\delta\phi(r) + 4\pi\delta(r), \tag{A.4}$$

where

$$k_s^2(z) \approx 4\pi \sum_k \delta(\epsilon(k) - \mu)|f_k(z)|^2 \tag{A.5}$$

in the low-temperature limit. Taking the Fourier transform of equation (A.4) with respect to the components of r parallel to the surface, it becomes

$$d^2\,\delta\phi(k_\parallel, z)/dz^2 = -(k_s^2(z) - k_\parallel^2)\,\delta\phi(k_\parallel, z) + 4\pi\delta(z - z_0), \tag{A.6}$$

where $\delta\phi(k_\parallel, z)$ is the Fourier transform on $\delta\phi(r)$ over the coordinates parallel to the surface. Let us attempt to find a solution to equation (A.6) of the form $\delta\phi = e^{-S(z)}$. Substituting in equation (A.6) we obtain

$$[(dS/dz)^2 - d^2S/dz^2 - (k_s^2(z) - k_\parallel^2)]e^{-S} = 4\pi\delta(z - z_0). \tag{A.7}$$

If typical values of the screening length are small compared to the width of the surface region, we can for most z neglect d^2S/dz^2 compared to $(dS/dz)^2$. With this approximation, we obtain a solution

$$\delta\phi = (4\pi q/2k_s'(z_0)) \exp\left[-\left|\int_{z_0}^z k_s'(k')\,dz'\right|\right], \tag{A.8}$$

where $(k_s')^2 = k_s^2 - k_\parallel^2$. In the extreme limit in which typical values of $k_s'^{-1}$ are much smaller than the width of the surface region, we can replace the integral in the exponent by $k_s'(z_0)(z-z_0)$ to a good approximation. The resulting form for $\delta\phi$ gives an inverse Fourier transform on $k_\parallel$ proportional to

$$\frac{e^{-k_s(z_0)|\boldsymbol{r}-z_0\hat{z}|}}{|\boldsymbol{r}-z_0\hat{z}|}, \tag{A.9}$$

the form of the result for Thomas–Fermi treatment of the potential due to a point charge in the bulk of the metal. While the extreme limit of screening length much smaller than the width of the surface region is not likely to occur, we do not expect the screening in real situations to be so qualitatively different to the bulk Thomas–Fermi screening. Although there is likely to be some anisotropy [16], one would not expect the screening that occurs parallel to the surface to be qualitatively different to that normal to the surface. This is illustrated in figures 3(a) and (b) of [16], which give the results of a local density approximation calculation of the electron charge distribution around a proton placed at two different locations in the surface region of a model for a metal. As can be seen in figure 3(b), even when the proton is located well out of the surface region (i.e., almost in the vacuum), the charge distribution still appears to be quite symmetrical.

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